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Electrostatic Field in the Trail of Ionospheric Satellites

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Conceptual similarities between the electrostatic near wake of an ionospheric satellite and the fluid-dynamic near wake of a blunt-base body in low-speed flow are noted. A simplified model of the electrostatic wake is constructed accordingly. There are recognized distinct regions where different approximate solutions of the Poisson equation are valid, namely: 1) an outer region, conceptually analogous to the external inviscid flow, where the self-induced electric field due to the charge separation develops independently of the boundary conditions on the body; 2) an inner region, analogous to the recirculation zone, where the electric field configuration is largely governed by the vehicle shape, whereas the field intensity is controlled by the flight condition (the ion Mach number) and by the difference between the body floating potential and the minimum potential characteristic of the flow; 3) an intermediate region, analogous to the fluid-dynamic free shear layer, where a rapid adjustment between the two aforenoted solutions is attained. Analyses appropriate to the various regions are presented and their combination to obtain a composite solution is described. The merit of the model resides in its ability to provide rapid estimates of the electrostatic wake; its validity is corroborated by heuristic arguments as well as by comparisons with the results of numerical solutions for axisymmetric configurations.

I. Introduction

A DETAILED knowledge of flowfields around ionospheric satellites is required in many data reduction problems, e.g., the interpretation of on-board measurements and of radar observations, the calculation of drag, etc. The gross features of the flow can easily be identified by considering the set of parameters characteristic of the physical processes in the plasma, by comparing their relative magnitudes in the situation of interest, and by determining accordingly the dominant physical effects. However, detailed analysis remains rather elaborate, particularly with regard to the charged components of the gas in the trail where self-induced electric fields and externally imposed magnetic fields dominate the behavior of charged particles over regions having extent comparable to or larger than the typical vehicle dimension.

The interaction between satellites and charged particles in the ionosphere has received considerable attention in recent years[1,2]; as a result, the roles of electrostatic and magnetostatic fields are understood. For typical ionospheric conditions (altitudes between 150 and 1500 km) and typical vehicle dimensions of the order of meters, the effect of the magnetic field is manifested mainly in the structure and decay of the far trail,[1,3,4] whereas the effect of the electric field is confined to a thin Debye sheet on the windward side of the vehicle, and to a near trail region, hereafter called the electrostatic wake, on the leeward side.[1,5–7] The transversal scale of the electrostatic wake is comparable to the typical body dimension a, while its streamwise scale is of the order of $M_i a$, M_i being the ion Mach number.

The present paper is concerned with the electrostatic wake in the context of the simplified over-all description of the trail initiated in Refs. 3 and 4. Specific attention is devoted to the extension of previous physical-analytical models[1,7] with a view toward obtaining improved agreement between approximate analysis and the results of numerical solutions of the governing equation[5,6] (the Poisson equation). The formulation of the model is guided by several conceptual similarities between the present problem and the fluid-dynamic near wake downstream of a blunt-base body in low-speed flow. In this vein the electrostatic wake is divided in distinct regions where different approximate solutions of the Poisson equation are valid. Broadly, the regions represent the counterparts of the external inviscid flow, the free shear layer and the recirculation zone.

The paper touches on various aspects of the investigation in the following sequence: upon a brief restatement of the problem and a heuristic justification of the model (Sec. 2),

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the approximate solutions appropriate to the various regions are set forth, and their combination into a composite description of the flow is presented (Sec. 3). Finally, the accuracy of the proposed model is assessed by comparing the present results with those obtained by numerical solution of the Poisson equation over the entire electrostatic wake.[6]

II. Formulation of the Model

The parameters characteristic of the physical processes in the plasma flow around an ionospheric satellite are: the flight velocity $\mathbf{U}$, the particle mass m_j, the mean thermal speed $\bar{c}_j$, and the mean free path L_j of each component j present, the electrostatic potential Φ_b on the surface of the vehicle, the Debye length $\lambda = (kT_e/4\pi e^2 n_e)^{1/2}$, and the Larmor radii $\rho_{Hj} = (c/eB_o)(2m_j kT_j)^{1/2}$ for the charged species. Under representative conditions, viz. a vehicle having a typical dimension a of the order of meters and orbital altitude between 150 and 1500 km, the relative magnitudes of these parameters are (subscripts e, i, and n refer to electrons, ions, and neutrals, respectively)

$$\lambda \leq \rho_{H_e} \ll a \leq \rho_{H_i} < L_{e,i} \leq L_n \tag{1a}$$

$$\bar{c}_n \approx \bar{c}_i \ll U \ll \bar{c}_e \tag{1b}$$

$$m_i U^2/2 \gg e\Phi(\mathbf{x}) \approx kT_i \approx kT_e \tag{1c}$$

over a major portion of the flow. Since the velocity of the satellite is hypersonic relative to the ions and subsonic relative to the electrons there develops a region of the lee side on the vehicle where the ion density falls drastically below the freestream value, while the electron density, in the absence of electric fields, tends to remain essentially undisturbed. Obviously, an electric field arises due to charge separation; the region in question is then identified with the electrostatic wake of interest here (Fig. 1). In view of the condition Eq. (1c) the electric field in the near wake has no appreciable effect on the motion of the ions but a dominant influence on the motion of the electrons; its presence tends to restore local charge neutrality in the flow over a scale dictated by the local Debye distance and approximately bounded by the magnitude of the transversal body dimension. Due to the extreme rarefaction of the ions, the electrostatic field is characterized by large negative values of the potential; the minimum is of such magnitude that the electron density becomes depleted to the level where the local Debye distance approaches the bound $\lambda \approx a$. If the electrons are assumed in equilibrium, viz. $(n_e/n_{e_\infty}) = \exp(e\Phi/kT)$, the minimum potential is then given by

$$\Phi_{\min} \approx -(kT/e)\ln(a/\lambda_\infty)^2 \tag{2}$$

For the representative conditions defined at the beginning of this section, $\ln(a/\lambda_\infty)^2 \approx 10$ and $|\Phi_{\min}| \approx 1$ to 2 v.

The estimate Eq. (2) was set forth in the early work of Gurevich,[1] who, in fact, used it to arrive at a two-region model of the electrostatic wake. According to this model the inner region is bounded by the body surface and by the equipotential line (or surface) $\Phi = \Phi_{\min}$; the outer region extends downstream of the equipotential line $\Phi = \Phi_{\min}$. Quasi-neutrality of charge, as well as equilibrium of the electron gas, are assumed to prevail in the outer region; the boundary $\Phi = \Phi_{\min}$ and, in fact, the potential field throughout that region are determined from the a priori known ion density field (dependent on the ion Mach number but not on the electric field). Charge density is assumed negligible in the inner region; the solution there is obtained by solving the Laplace equation subject to the prescribed boundary conditions on the surface of the body and on the equipotential $\Phi = \Phi_{\min}$. The approach leads to curves of potential vs downstream distance in the wake having physically unreasonable shape and discontinuous slope at the interface between the two regions; moreover, the estimated minimum potential is in error by approximately 50%.[6]

It has been suggested in Ref. 6 that the failure of the Gurevich model is caused by the patching process itself. A different view is taken in the present paper, namely, the electrostatic wake can be described in terms of inner and outer regions provided improved approximations are used in their description and in the definition of their common boundary. A similar approach has previously been pursued in Ref. 7 for the limit case of $M_i = \infty$ flow over a circular disk with $(\lambda_\infty/a) \ll 1$. In the absence of thermal speed and of scattering of the ions in the electric and magnetic fields, Greifinger[7] arrived at a simplified picture of the trail as a cylindrical hold (having the same cross section as the body) devoid of ions and low in electron density, except for a sheath extending a few Debye lengths λ_∞ about the cylindrical boundary. The potential field in a major portion of the hole is one-dimensional and independent of the boundary conditions on the body surface; these are accommodated in a neighborhood of the surface, within a downstream distance comparable to the body transversal dimension.

The brief review clearly indicates how the Guervich and Greifinger models differ in the criteria for the approximate description of various flow regions and for the matching (or patching) of the corresponding solutions. Specifically, the following features can be identified in the Greifinger model: 1) the assumption of quasi-neutrality is abandoned even in the outer portion of the electrostatic wake; 2) the inner limit of the one-dimensional sheath solution provides the boundary condition for the problem in the inner region, where accommodation to the body potential is attained; however, the two solutions are not joined on an equipotential line.

These features possess obvious physical appeal inasmuch as: 1) appreciable charge separation is prerequisite to the build-up of a large electrostatic field; 2) the free sheath must merge with the attached sheet that prevails on the windward side of the body; accordingly, the potential on the interface with the inner region must vary between the minimum value in the flow and the value on the body.

Extension of these ideas to flows at large, but finite, Mach number M_i, and large ratios (a/λ_∞), leads to a picture of the electrostatic wake as a conical cavity with axis in the streamwise direction, base approximately coincident with the body cross section, and height of the order $M_i a$ (Fig. 1).

A free sheath (outer region) extends inward from the conical boundary of the cavity; its structure is governed mainly by the self-induced electrostatic field which, in the absence of strong body bias, develops consistent with the prescribed ion density (essentially unaffected by the field) and with the electrons in local equilibrium.

The perturbing influence of the body potential is manifested over an inner region having dimensions comparable to the body dimension a. The solution for the electrostatic potential in that region is determined by the boundary condition on the surface of the body as well as by the requirement that it match with the sheath solution. Thus, a heuristic definition of the matching criteria is necessary for a full description of the model; this can be obtained by a qualitative investigation of the solutions of the Poisson equation written in dimensionless form [with a as length scale, (kT_{e_∞}/e) as potential scale, and $n_{i_\infty} = n_{e_\infty}$]

$$\nabla^2\varphi = -(a/\lambda_\infty)^2[n_i/n_{i_\infty} - n_e/n_{e_\infty}] \tag{3a}$$

If, in a first approximation, the electrons are assumed in equilibrium Eq. (3a) becomes

$$\nabla^2\varphi = -(a/\lambda_\infty)^2[n_i/n_{i_\infty} - e^{\varphi}] \tag{3b}$$

For $(a/\lambda_\infty) \gg 1$ the solutions of Eq. (3b) follow a different behavior in the region of the wake where the quantity $A = (a/\lambda_\infty)^2(n_i/n_{i_\infty}) \gg 1$ as opposed to the region where $A \ll 1$. For $A \gg 1$ the solution is approximated by[1]

$$\varphi \approx \ln(n_i/n_{i_\infty}) \tag{4a}$$

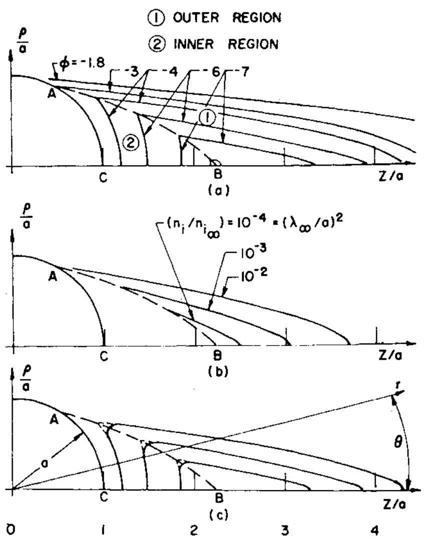

Fig. 1 Schematic structure of the electrostatic near wake; sphere, $M_i = 8$, $\lambda_\infty/a = 1/100$, $\varphi_b = -3$. a) juxtaposition of inner and outer solutions; b) approximate boundary between inner and outer regions; c) comparison between patched and composite solution.

while for $A \ll 1$

$$\varphi \approx -\ln(a/\lambda_\infty)^2 \approx \varphi_{\min} \tag{4b}$$

A direct comparison of Eqs. (4a) and (4b) underlies the Gurevich criterion whereby transition between the two approximations is assumed to occur on the constant ion density line $(n_i/n_{i_\infty}) \approx (\lambda_\infty/a)^2$. As noted previously, this view requires modification. The approximate revisions are suggested by the following observations.

1) The interface between the inner and outer regions (i.e., the boundary between the domains of validity of the two approximate solutions of the Poisson equation) must be anchored to the body near the point where the outer solution predicts $\varphi = \varphi_b$, and to the axis near the point where the quantity $A_e = (a/\lambda_\infty)^2(n_e/n_{e\infty})$ approaches unity. Transition near the axis is identified by the criterion $A_e \approx 1$, in lieu of the criterion $A = 1$ suggested by Gurevich, because charge separation, specifically negative net charge density $(n_e/n_{e_\infty}) > (n_i/n_{i_\infty})$, prevails in the electrostatic wake. The two criteria coincide, i.e., the interface becomes anchored to the line $(n_i/n_{i_\infty}) = (\lambda_\infty/a)^2$ near the axis, only for conditions of moderate charge separation, viz. $A \approx A_e$ in the outer region. Clearly, under these conditions the distance between the body surface and the line $(n_i/n_{i_\infty}) = (\lambda_\infty/a)^2$ must remain smaller than, or equal to, the maximum estimated extent of the inner region, viz. two or three typical body dimensions. Since the ion density field is essentially defined by the ion Mach number M_i, it follows that the inner and outer solutions are properly joined in a neighborhood of the line $(n_i/n_{i_\infty}) = (\lambda_\infty/a)^2$ only when $M_i \leq M_{ie}$. For $M_i \geq M_{ie}$, in particular $M_i \to \infty$, and for a fixed body potential, the solution in the inner region becomes independent of M_i. The upper bound M_{ie} is defined as the flight condition wherein the outer solution obtains its asymptotic value on or about the considered constant ion density line.

2) Additional information about the geometry of the interface can be gained from a qualitative study of the inner solution. In this connection it is observed that charge density is negligible in the inner region, and, because of charge shielding, the boundary conditions on the interface can influence the inner solution only in a domain extending a few Debye lengths‡ in direction normal to the interface. The lateral extent of this domain varies greatly as a function of position along the interface; only near the axis (where $\varphi \approx \varphi_{\min}$ and $A_e \approx 1$) does it become comparable to the dimension of the inner region. Hence, the inner solution must be dominated by the boundary condition $\varphi = \varphi_b$ on the surface of the body, and by the conditions imposed on the portion of the interface near the axis. The dependence is satisfied by an inner solution one-dimensional in body oriented coordinates; the associated interface is approximately described as the locus of points where lines of equal potential in the inner and outer regions do intersect.

3) The one-dimensional inner solution can easily be determined case by case; for example, in the case of a spherical body at $\varphi = \varphi_b$ the potential is described by

$$\varphi = \varphi_b + b[a/r - 1][\varphi_{\min} - \varphi_b] \tag{5}$$

with b as constant to be determined upon joining of inner and outer solutions on the axis.

In summary, the following model and analysis of the electrostatic wake have been evolved heuristically: for $(a/\lambda_\infty) \gg 1$ and $M_i \gg 1$ an inner and outer region can be recognized (Fig. 1). The outer region consists of a free sheath which can be analyzed independent of the boundary conditions on the surface of the body. For Mach numbers M_i not exceeding a bound M_{ie} the inner region may approximately be described by a one-dimensional solution in body oriented coordinates. The scale of this solution is determined by imposing the value of the potential (specifically the asymptotic value of the sheath solution) at the point where $(n_i/n_{i_\infty}) = (\lambda_\infty/a)^2$ on the axis. For $M_i \geq M_{ie}$ and φ_b constant, the inner solution remains invariant, and identical with that for $M_i = M_{ie}$; the latter solution is determined by the condition that the outer potential attains its asymptotic value at $(n_i/n_{i_\infty}) = (\lambda_\infty/a)^2$. The approximate boundary between inner and outer regions is defined as the locus of points where lines of equal potential in the two regions do intersect (Fig. 1). The approximate description of the electrostatic wake so obtained represents the actual solution except for minor adjustments in the immediate neighborhood of the boundary defined previously (Fig. 1).

The present model does not include the effects of departures from equilibrium of the electron gas. Certainly these effects are present in the inner region as well as in a portion of the outer region adjacent to the inner. Careful analysis would require simultaneous solution of the Poisson equation and of the kinetic equation for the electrons, taking into account the two-sided nature of the distribution function in the inner region.

At this point and in this context it is of interest to note an analogy between the present problem and that of a fluid dynamic wake downstream of a blunt-base body in low-speed flow. Consideration of the analogy can assist the fluid dynamicist in the formulation of analyses that account for nonequilibrium distribution of the electrons. At given $M_i \gg 1$, $(a/\lambda_\infty) \gg 1$, and body configuration, the solution for the electrostatic wake, neglecting departures from equilibrium for the electrons, represents the counterpart of the

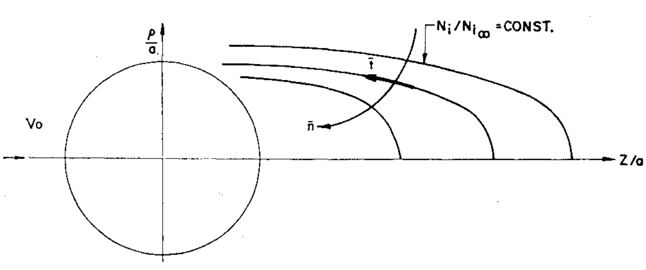

Fig. 2 Coordinate system used in the analysis of the outer region.

‡ Although electrons are moderately out of equilibrium in the neighborhood of the interface, the Debye length computed under the assumption of equilibrium provides a suitable scale for shielding effects.

infinite Reynolds number solution for the fluid-dynamic near wake downstream of a given configuration. In this context, the sheath plays the role of the external inviscid flow, the inner region the role of the recirculation zone, and the interface between the two regions the role of the free shear layer. The downstream condition is one of prescribed decay for both wakes. Departures from local equilibrium distribution for the electrons have effects on the electrostatic wake similar to those of finite, albeit large, Reynolds number on the fluid-dynamic wake. The potential at the interface between the two regions is decreased in line with the electron density; the region of adjustment between the inner and outer solutions becomes broader as electrons out of equilibrium influence the tail of the potential curve in the sheath; the solution for the two regions must be obtained in iterative fashion since the electron density in the region of matching becomes dependent on the solution over the entire domain.

Analyses of fluid-dynamic near wakes are rather unwieldy; in view of the analogy the same situation can be expected in the present problem. Thus, it becomes of interest to assess the merits of the approximate model presented above. With this objective, we proceed to present an outline of the calculations required.

III. Analytical Statement of the Model

The analysis of the outer region (sheath) is conveniently performed in a system of curvilinear orthogonal coordinates $(\bar{n},\bar{t},\bar{\theta})$ oriented along and normal to the surfaces of constant ion density $\eta = (n_i/n_{i_\infty})$ (Fig. 2). For axisymmetric problems, the Laplacian operator in $(\bar{n},\bar{t},\bar{\theta})$ coordinates is

$$\nabla^2 = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial \bar{t}}\left(\frac{h_2h_3}{h_1}\frac{\partial}{\partial \bar{t}}\right) + \frac{\partial}{\partial \bar{n}}\left(\frac{h_1h_3}{h_2}\frac{\partial}{\partial \bar{n}}\right)\right] \quad (6a)$$

where

$$h_1^2 = (\partial\rho/\partial\bar{t})^2 + (\partial z/\partial\bar{t})^2 \quad (6b)$$

$$h_2^2 = (\partial\rho/\partial\bar{n})^2 + (\partial z/\partial\bar{n})^2 \quad (6c)$$

$$h_3 = \rho \quad (6d)$$

Since Debye shielding dominates in the sheath the local Debye length $\lambda = \lambda_\infty(n_{e_\infty}/n_e)^{1/2}$ represents the proper length scale in the $\bar{n}$ direction; accordingly new independent variables (n,t) are introduced

$$n = \int_0^{\bar{n}}\left(\frac{a}{\lambda}\right)h_2 d\bar{n} \qquad t = \bar{t} \quad (7)$$

with $n = 0$ well within the sheath§ and the Poisson equation (3b) is recast in the form

$$\frac{\partial^2\varphi}{\partial n^2} + \left[\frac{1}{2}\frac{\partial\varphi}{\partial n} + \frac{1}{\rho h_1}\frac{\partial}{\partial n}(\rho h_1)\right]\frac{\partial\varphi}{\partial n} + \left(\frac{\lambda}{a}\right)^2\frac{1}{\rho h_1 h_2}\frac{\partial}{\partial t}\left(\frac{\rho h_2}{h_1}\frac{\partial\varphi}{\partial t}\right) = 1 - \eta e^{-\varphi} \quad (8)$$

In the sheath $(\partial\varphi/\partial n) = 0(1)$, $(\lambda/a)^2 \ll 1$; also $\partial ln(\rho h_1/\partial n) = 0(1/M_i) \ll 1$ for ionospheric satellites (Fig. 3). Accordingly Eq. (8) can be reduced to the ordinary differential equation [a superscript (o) denotes outer solution]

$$\frac{d^2\varphi^{(o)}}{dn^2} + \frac{1}{2}\left(\frac{d\varphi^{(o)}}{dn}\right)^2 = 1 - \eta e^{-\varphi^{(o)}} \quad (9)$$

subject to boundary conditions at $n \to \pm\infty$. The boundary condition at $n \to -\infty$ enforces matching with the quasi-neutral solution of Gurevich,[1] viz.

$$n \to -\infty \quad \varphi^{(o)} \to \ln\eta \quad (10)$$

§ In the calculations reported here the line $n = 0$ was defined as the locus of points where the steepest tangent to the actual ion density distribution predicted $\eta = 0$.

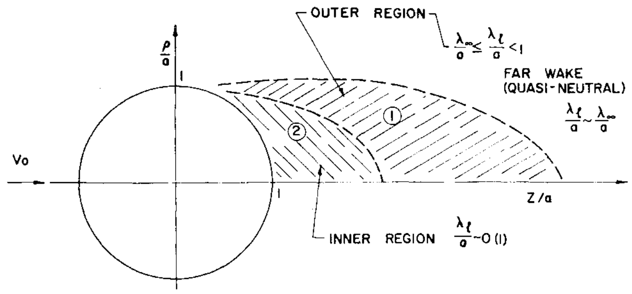

Fig. 3 The magnitude of the shielding distance in the subregions of the electrostatic wake.

The boundary condition at $n \to \infty$ is readily obtained by recognizing that $(\eta e^{-\varphi}) \ll 1$ at the inner edge of the sheath; accordingly Eq. (9) there takes the asymptotic form

$$\frac{d^2\varphi^{(o)}}{dn^2} + \frac{1}{2}\left(\frac{d\varphi^{(o)}}{dn}\right)^2 = 1 \quad (11)$$

with the integral

$$d\varphi^{(o)}/dn = -(2)^{1/2}\tanh[n/(2)^{1/2} + c] \quad (12)$$

c being a constant of integration.

Thus, the second boundary condition for Eq. (9) is

$$n \to \infty \quad (d\varphi^{(o)}/dn) = -(2)^{1/2} \quad (10b)$$

The ion density η appearing in Eq. (9) must be determined consistent with the body cross section shape and with the ion Mach number for M_i. For axisymmetric bodies the solution for a circular disk (or body of circular cross section) set normal to a hypersonic stream $(M_i \gg 1)$ can be employed[1]

$$\left(\frac{n_i}{n_{i_\infty}}\right) = \eta = 1 - 2\exp\left(-\frac{\rho^2 M_i^2}{z^2}\right) \times \int_0^{M_i a/z} \tau\exp(-\tau^2)I_o\left(\frac{2\rho M_i\tau}{z}\right)d\tau \quad (13)$$

for other configurations (e.g., two-dimensional or fully three-dimensional bodies) solutions equivalent to Eq. (13) must be determined.

Upon substitution of Eq. (13), or equivalent, into Eq. (9), the latter may be integrated numerically. The quasi-linearization technique[8] has been applied and found to be satisfactory in the calculation reported here. The boundary conditions strictly applicable at $n \to \pm\infty$ have been imposed at finite n (say $n = \pm n^*$); numerical experiments have shown that the solutions are unaffected if $|n^*| \geq 3$. The integration of Eq. (9) is carried out in the (n,t,φ) space; the inverse transformation Eq. (7) to $(\bar{n},\bar{t},\varphi)$ variables is performed subsequently. Typical results of a sheath solution are exhibited in Fig. 4. The reduction of Eq. (9) to an ordinary differential equation represents a substantial simplification in the analysis of the outer region; this becomes especially significant when fully three-dimensional situations are considered.

The solution in the inner region may now be determined readily. The position z^* of the matching point is obtained

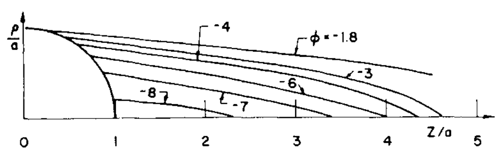

Fig. 4 The outer solution for the electrostatic potential.

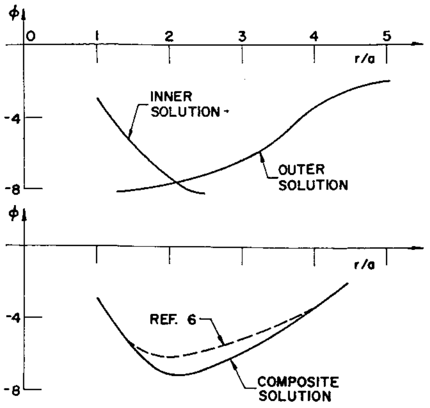

Fig. 5 Typical trends of inner, outer and composite solution; sphere, $\theta = 4.5°$, $M_i = 8$, $\lambda_\infty/a = 1/100$.

from Eq. (13)

$$(n_i/n_{i_\infty}) = (\lambda_\infty/a)^2 \text{ at } z/a = z^*/a = M_i/[2 \ln(a/\lambda_\infty)]^{1/2} \quad (14)$$

If, correct to logarithmic terms of order $[2 \ln(a/\lambda_\infty)]$, charge density is neglected in the inner region, the one-dimensional solution Eq. (5) can be used to determine the potential. In the specific case of a spherical body of radius a, Eq. (5) becomes

$$\varphi^{(i)} = \varphi_b + \left[\frac{a}{r} - 1\right] [\varphi_{\min}{}^{(o)} - \varphi_b] \left[\frac{\{2 \ln(a/\lambda_\infty)\}^{1/2}}{M_i} - 1\right]^{-1} \quad (15)$$

A typical solution, Eq. (15), is juxtaposed with the associated outer solution in Fig. 1; part c of the figure exhibits the small adjustment required to reconcile the straightforward model with a composite solution. The adjustment can only be determined by numerical solution of the Poisson equation (including an appropriate description for the electron density) in a domain extending several (≥ 3) local Debye lengths on either side of the line of demarcation $r = r^*(\theta)$ between inner and outer regions; the solution $\varphi^{(a)}$ (r,θ) is determined by the conditions that it match the normal component of electric field dictated by the inner (outer) solution on the aforenoted line of demarcation, and it satisfies the condition of symmetry on the axis. A typical result of this process is shown in Fig. 5. The composite solution $\varphi^{(c)}$ of Fig. 5 is obtained by parroting the standard rules of matched asymptotic expansion techniques.

The results of separate numerical integrations in the inner and outer regions have been tested against, and found to be in agreement with, the results of numerical solutions for the entire electrostatic wake, using the same assumptions for the electron density in the region of presumed nonequilibrium. In view of this agreement, additional calculations and comparisons have been instituted.

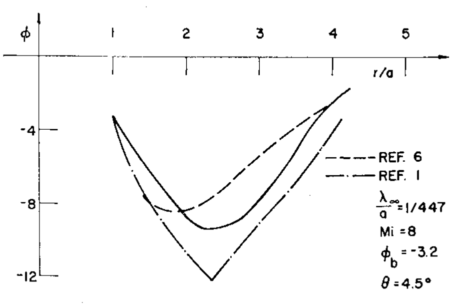

Fig. 6 Comparison of present solution with Refs. 1 and 6.

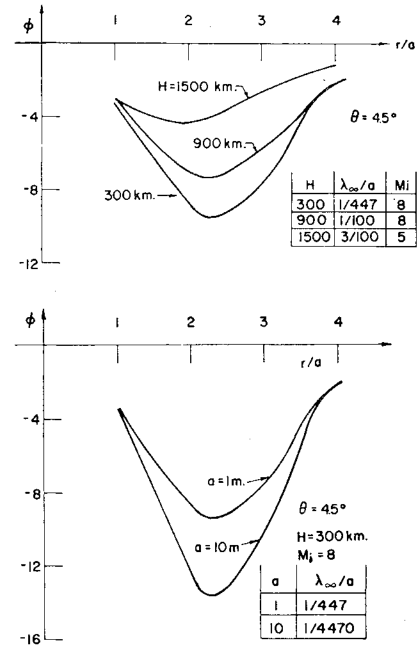

Fig. 7 Typical dependence of solution upon λ_∞/a and M_i.

Results and Discussion

Calculations have been carried out with a view toward comparisons with the results of other authors, more extensive support for the straightforward model (one-dimensional inner solution), and quantitative demonstration of the influence of the parameters M_i and (λ_∞/a) upon the extent of, and minimum potential in, electrostatic wakes.

Figure 6 presents a comparison between the predictions of the three separate analyses (Refs. 1, 6, and the present one) for a specified configuration and flight condition. Comparisons have been instituted between the predictions of the present analysis and those of Refs. 1 and 6 for spherical bodies. The present results have been obtained under the assumption of: 1) ion density distribution described by Eq. (13), which is strictly appropriate to a disk; 2) electrons in equilibrium in the outer region i.e., $(n_e/n_{e_\infty}) = \exp(\varphi)$; 3) electron density in the inner region either zero or constant, $(n_e/n_{e_\infty}) = \exp(\varphi_{\min})$, along radial lines.¶

By comparison, the results of Ref. 6 were obtained under the assumption of: 1) ion density coincident with the exact neutral density distribution around a sphere; 2) electron density distribution along radial lines in the outer region (outside the potential barrier) described by the solution for an electrostatic probe; and 3) constant electron density along radial lines in the inner region, $(n_e/n_{e_\infty}) = \frac{1}{2}\exp(\varphi_{\min})$. Finally, the predictions of Ref. 1 were obtained from Eqs. (13, 4a, and 4b).

The different approximations for ion density used in Ref. 6 and in the present paper are not expected to influence the comparison insomuch as their differences are appreciable only in a neighborhood of the body surface where the effect of ions becomes negligible. In contrast, the different assumptions for electron density at the potential crest, $(n_e/n_{e_\infty}) = \exp(\varphi_{\min})$ here as opposed to $[\frac{1}{2}\exp(\varphi_{\min})]$ in Ref. 6, are expected to influence the length scale, but not the behavior, of the inner solution. The point of view is substantiated by the following discussion of results. It is seen that the present prediction for the minimum potential is in reasonable agreement with that of Ref. 6, but appreciably higher than

¶ Clearly $(n_e/n_{e_\infty}) = 0$ in the straightforward model. The assumption $(n_e/n_{e_\infty}) = \exp(\varphi_{\min}) \neq 0$ has been used only in comparative numerical analysis of the inner region.

that of Ref. 1. There is disagreement with Ref. 6 on the position of minimum potential; however, this can be entirely attributed to the different assumptions for the (φ, n_e) relationship in the two calculations. In accord with the view set forth here, the distances between the point of minimum potential and the body surface are almost in the exact ratio of the maximum Debye lengths computed in the two cases.

Also, in either case the potential in the inner region is closely approximated by a one-dimensional solution of the Laplace equation; thus the effect of charge density in the inner region may indeed be neglected with little loss of accuracy. In all cases, the model has provided predictions for the potential within 10% of those obtained by the composite solutions.

Figure 7 exhibits the effect of changing Mach number and ratio (λ_∞/a) by considering typical flight conditions at various altitudes H. In line with the prediction of the model, the position of minimum potential moves closer to the body as the Mach number decreases; the magnitude of the minimum potential increases as the ratio (λ_∞/a) decreases.

In conclusion, there is reasonable support for the simple model and its capability to provide predictions of satisfactory engineering accuracy. It should be remarked that considerable savings of numerical labor have been obtained; in particular, the elimination of the iterative processes and the mesh selection problems usually associated with fully numerical schemes. Significant advantages should be accrued in fully three-dimensional situations.

It should also be pointed out that an external magnetic field will cause the trail to assume a structure[3,4] which involves "spots" of electrostatic wake as vestiges of the near flowfield cyclically reproduced downstream. The simplified model analysis should also prove useful in the repetitive analysis of these "spots."

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